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A Study on the Performances of Multivariate Exponentially Weighted Moving Average (MEWMA) and Multivariate Synthetic Charts

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ABSTRACT

A multivariate control chart is a common tool used for monitoring and controlling a process whose quality is determined by several related variables. The objective of this study is to compare the performances of the multivariate exponentially weighted moving average (MEWMA) and the multivariate synthetic T^2 control charts, for the case of a multivariate normally distributed process. A comparative study is made based on the average run length (ARL) performances of the control charts, using the simulation method, in order to identify the chart having the best performance in monitoring the process mean vector. The performances of the two charts, for different sample sizes and correlation coefficients, are presented in this paper. It was found that the MEWMA chart outperformed synthetic T^2 chart for small shifts but the latter prevailed for moderate shifts. Both charts performed equally well for larger shifts. In addition, the performances of both MEWMA and synthetic T^2 charts were found to be influenced by sample size and correlation coefficient. The two charts' performances improved as the sample size and correlation coefficient increased for small and moderate shifts, but the charts' performances did not depend on sample size and correlation coefficient when the shift was large.

Keywords: Average run length (ARL), MEWMA chart, multivariate synthetic chart, out-of-control, in-control

INTRODUCTION

Statistical process control (SPC) is used to describe a set of problem solving tools that have been used to measure and analyze variations in processes. SPC can be implemented by common key monitoring and investigation tool, which is the control chart.

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It is a statistical tool used to detect excessive process variability due to specific assignable causes that can be corrected (Shewhart, 1986).

Univariate control charts involving the measurement of a single variable include the popular Shewhart $\overline{X}-R$ and $\overline{X}-S$ charts, as well as the moving average and moving range charts. When more than one variable is collected, the relationship between several different variables need to be shown, resulting in multivariate charts such as Hotelling's T^2, multivariate exponentially weighted moving average (MEWMA), and multivariate cumulative sum (MCUSUM) charts (Montgomery, 2009).

Ghute and Shirke (2008) and Lee and Khoo (2006) suggested two approaches on multivariate synthetic and MEWMA charts. Lee and Khoo (2006) also proposed an optimal statistical design of the MEWMA chart based on the average run length (ARL) and median run length (MRL) criteria. The optimal design of the MEWMA chart was made simple by using plots of the optimal smoothing constant r and plots of the optimal control limit, H. The constructed plots give approximation of the optimal r and its corresponding H for the given in-control ARL and MRL. Meanwhile, Ghute and Shirke (2008) proposed a multivariate synthetic control chart, which is a combination of the Hotelling's T^2 chart and the conforming run length (CRL) chart for monitoring the mean vector of a multivariate normally distributed process.

Some of the recent works on the multivariate synthetic charts were done by Khoo *et al*. (2009), Khoo *et al*. (2011), and Aparisi and de Luna (2009), while those on the MEWMA charts were carried out by Niaki and Ershadi (2012), Alkahtani and Schaffer (2012), Xie *et al*. (2011), Alipour and Noorossana (2010), and Khoo and Quah (2004).

The objective of this study was to compare the ARL performances of the optimal MEWMA chart and the optimal multivariate synthetic T^2 chart. In the optimal design of the MEWMA chart, the method proposed by Lee and Khoo (2006) is used. Note that the optimal multivariate synthetic T^2 chart was proposed by Ghute and Shirke (2008). ARL, which is the expected value of the run length, is used to measure the performance of the control charts. When comparisons were made, the charts were first designed to have a common in-control ARL and then the out-of-control ARLs for a given shift in the process were compared. The common in-control ARL values of the charts were set arbitrarily by the investigator. The chart with a better performance is the chart with a smaller out-of-control ARL.

In the next section, the MEWMA and the multivariate synthetic T^2 charts are reviewed. This is followed by a comparison between the performances of MEWMA and the multivariate synthetic T^2 charts for different sample sizes and correlation coefficients through a simulation study. Finally, conclusions and some suggestions for further research are presented.

MULTIVARIATE CONTROL CHARTS

The MEWMA Chart

The multivariate Shewhart type control chart, like the Hotelling's T^2 chart, uses the information from the current sampling and is insensitive to small and moderate shifts in the mean vector. The MEWMA control chart is one of the multivariate control charts developed to solve this problem (Bersimis *et al*., 2007).

The MEWMA chart proposed by Lowry *et al*. (1992) is a logical extension of the univariate exponentially weighted moving average (EWMA) chart. The MEWMA statistics are written as follows:

$$Z_i = r\overline{X}_i + (1-r)Z_{i-1}, \quad i = 1, 2, \ldots, \tag{1}$$

where Z = 0 and $0 < r \le 1$. Note that Z_0 is an initial vector and r is a smoothing constant. The multivariate observations $\overline{X}_1, \overline{X}_2, \ldots$, are assumed to be independently and identically distributed multivariate normal random vectors, each with p quality characteristic. It is assumed that μ_0 is the in-control process mean vector of zeros and Σ is the covariance matrix. The multivariate control charts for the mean vector are designed to detect shifts over time from this in-control vector. The quantity plotted on the MEWMA chart is:

$$T_i^2 = Z_i' \textstyle\sum_{Z_i}^{-1} Z_i, \qquad i = 1, 2, \ldots, \tag{2}$$

where $\sum_{Z_i} = \{r[1-(1-r)^{2i}]/[(2-r)n]\} \sum$ is the covariance matrix of Z_i. The MEWMA chart gives an out-of-control signal as soon as $T_i^2 > H$, where H is the control limit which is a constant chosen to give a desired in-control ARL.

Lowry *et al*. (1992) have shown that the ARL performance of the MEWMA chart depends on the mean vector μ and the covariance matrix Σ only through the value of the non-centrality parameter δ, where,

$$\delta = \left(\mu' \textstyle\sum^{-1} \mu\right)^{1/2} \tag{3}$$

The Optimal MEWMA Control Chart

A MEWMA chart is optimal in detecting a shift if the MEWMA chart has the smallest out-of-control ARL among all MEWMA charts with the same in-control ARL for a particular shift. The optimal design specifies the optimal selection of the chart parameters, namely, the smoothing constant, r and its corresponding control limit, H for a MEWMA chart. Prabhu and Runger (1997) developed a design strategy for the MEWMA chart to obtain the optimal values of the chart's parameters. Lee and Khoo (2006) extended the optimal design approach of Prabhu and Runger (1997) for the MEWMA chart by recommending the four steps method in order to select the optimal chart parameters. In designing the optimal MEWMA chart, Lee and Khoo (2006) used the Markov chain method to select the control chart's parameters based on the desired size of a shift that must be detected quickly and the desired in-control ARL.

The Multivariate Synthetic Control Chart

The recent approaches in improving the performances of the univariate control charts include combining the Shewhart $\overline{X}$ control chart with a conforming run length (CRL) chart, leading to a synthetic control chart (Wu & Spedding, 2000). Ghute and Shirke (2008) extended the univariate synthetic chart for multivariate processes by developing the synthetic T^2 chart for monitoring the mean vector of a multivariate normally distributed process. This synthetic T^2 chart is a combination of the Hotelling's T^2 chart and the CRL chart.

Hotelling's T^2/S Sub-chart

The Hotelling's T^2/S sub-chart used in this paper is based on the Hotelling's T^2 chart that was used to construct the synthetic T^2 chart proposed by Ghute and Shirke (2008). The Hotelling's T^2 statistic is a generalization of the Student's t statistic used in the multivariate hypothesis testing. Consider the vectors representing the measurements of the p process characteristics, $X_1, X_2, ..., X_n$, as a random sample from the p – variate normal distribution. The mean vector is μ and the covariance matrix is Σ. μ_0 is the desired mean vector and Σ_0 is the desired covariance matrix. The intent is to detect the shifts in μ.

The Hotelling's T^2 charting statistic used in monitoring the p quality characteristics is given by:

$$T^2 = n(\overline{X} - \mu_0)' \sum_0^{-1} \overline{X} - \mu_0, \qquad [4]$$

where n is the sample size, $\overline{X}$ is the $(p \times 1)$ vector of sample means, and $\sum_0^{-1}$ is the inverse of the $(p \times p)$ covariance matrix.

The upper control limit is used to determine if a process is in-control or out-of-control based on the location of the plotted points taken from the sample statistics computed from independent subgroups of observations. The upper control limit takes the form:

$$UCL = \chi^2_{p,\alpha} \qquad [5]$$

where $\chi^2_{p,\alpha}$ is upper 100α percentage point of the chi-square distribution with p degrees of freedom and α is the probability of the Type I error for the T^2 control chart.

When the process is in-control, the plotted points lie below UCL, indicating that the T^2 statistic is distributed as a chi-square variable with p degrees of freedom. When the process is out-of-control, a plotted point lies above UCL, showing that the T^2 statistic is distributed as a non-central chi-square variable with p degrees of freedom and a non-centrality parameter, λ^2, given as:

$$\lambda^2 = n(\mu - \mu_0)' \sum_0^{-1} (\mu - \mu_0) = nd^2 \qquad [6]$$

The Mahalanobis distance, $d = \sqrt{(\mu - \mu_0)' \sum_0^{-1} (\mu - \mu_0)}$, is used to measure the change in the process mean vector.

The ARL for the Hotelling's T^2 chart can be calculated as:

$$ARL = \frac{1}{P} \qquad [7]$$

where $P = \Pr(T^2 > UCL | d)$. The on-target value of P can be determined as:

$$P(0) = \Pr(T^2 > UCL | d = 0) = 1 - F_p(UCL) \qquad [8]$$

where $F_p(x) = \Pr(\chi^2_p \leq x)$ is the cumulative distribution function of a central chi-square distribution with p degrees of freedom. The off-target value of P can be determined from:

$$P(d) = \Pr\left(T^2 > UCL \middle| d \neq 0\right) = 1 - F_{p,\lambda^2}(UCL) \quad [9]$$

where $F_{p,\lambda^2}(x) = \Pr(\chi_p^2 \leq x)$ is the cumulative distribution function of a non-central chi-square distribution with p degrees of freedom and non-centrality parameter, λ^2. Equations [4]-[9] are obtained from Ghute and Shirke (2008).

Conforming Run Length, CRL/S Sub-chart

The CRL chart was proposed by Bourke (1991) and it was originally developed for attribute quality control to detect shifts in the fraction non-conforming, θ. The CRL*/S* sub-chart employed in this paper is based on the CRL chart used to construct the synthetic T^2 chart proposed by Ghute and Shirke (2008).

Consider CRL which follows a geometric distribution with parameter, θ. Its expected value μ_{CRL} and the cumulative distribution function F_θ (*CRL*) can be calculated as follows:

$$\mu_{CRL} = \frac{1}{\theta} \quad [10]$$

$$F_\theta(CRL) = 1 - (1-\theta)^{CRL},\ CRL = 1, 2, ... \quad [11]$$

Equation [11] shows that the distribution of *CRL* changes with θ. The expected value of CRL decreases as θ increases and vice versa.

The CRL chart detects an increase in θ when $CRL \leq L$, where L is the lower limit required for the chart. The average number of the CRL samples required to detect an increase in θ is the average run length of the *CRL* chart, denoted as ARL_{CRL} (Wu & Spedding, 2000) and can be calculated as:

$$ARL_{CRL} = \frac{1}{F_\theta(L)} = \frac{1}{1-(1-\theta)^L} \quad [12]$$

The average number of the inspected units (ANS_{CRL}) required to signal a fraction non-conforming shift can be calculated as the product of Equations (10) and (12) shown below:

$$\begin{aligned} ANS_{CRL} &= \mu_{CRL} \times ARL_{CRL} \\ &= \frac{1}{\theta} \times \frac{1}{1-(1-\theta)^L} \end{aligned} \quad [13]$$

Equations [10] – [13] were obtained from Ghute and Shirke (2008).

The Optimal Multivariate Synthetic Control Chart

Ghute and Shirke (2008) developed a synthetic T^2 chart based on ARL. Let $ARL_s(d)$ denote the average number of the T^2 samples required for a synthetic T^2 chart to signal a shift of magnitude d in the mean vector. The $ARL_s(d)$ can be calculated from Equations [9] and [13] and is given as follows:

$$ARL_s(d) = \frac{1}{P(d)} \times \frac{1}{1-(1-P(d))^L} \quad [14]$$

where L is the lower control limit of the CRL/S sub-chart and $P(d)$ is the detecting power. The $ARL_s(0)$ should be large in order for the false alarm rate to be kept low. The $ARL_s(d)$ should be small so that the detection of the process can be made quickly. Suitable values of L and the upper control limit of the T^2/S sub-chart, UCL, must be obtained in order to design a synthetic T^2 chart. Ghute and Shirke (2008) solved the optimization problem and designed the synthetic control chart by minimizing the following out-of-control ARL:

$$ARL_s(d^*) = \frac{1}{P(d^*)} \times \frac{1}{(1-P(d^*))^L} \quad [15]$$

where d^* is the optimal shift, and a quick detection is needed.

RESULTS AND DISCUSSION

This paper involves a comparative study on the performances of MEWMA and the multivariate synthetic T^2 control charts based on the case of a multivariate normally distributed process. The Statistical Analysis System (SAS, version 9.1) software was used to conduct all the simulations in this paper.

A brief explanation on how the simulation was conducted is as follows: Firstly, samples of size $n \in \{5,10\}$ having bivariate normal observations (the number of variables, $p = 2$ is considered) are generated. Then, Equations [1] and [2] are used to compute the MEWMA statistics, while Equation [4] is used to compute the synthetic T^2 sub-chart's statistic. The number of samples required for each of the two charts to signal an out-of-control signal is recorded as the run length of the chart. The processes of generating samples with bivariate normal observations and recording the run length at each trial are repeated for 10000 trials for each of the charts. Finally, the average of the run lengths, based on the 10000 trials for each of the charts, is computed as ARL of the charts.

The out-of-control ARL performances of MEWMA and synthetic T^2 charts were compared for various sizes of shifts regarded as important, where the in-control ARL is set as 370. The ARL profiles for the optimal MEWMA and optimal synthetic T^2 control charts were also developed based on the sample sizes, $n \in \{5,10\}$, the number of variables, $p = 2$, design shifts, $d^* \in \{0.5,1.0\}$, and correlation coefficients, $\rho \in \{0.2,0.5,0.8\}$. Note that due to cost considerations, in practice, a large sample size is rarely used in process monitoring. For this reason, only small and moderate sample sizes, i.e. $n \in \{5,10\}$, were considered in this study. Thus, it should be pointed out that considering other sample sizes would give similar results as the MEWMA and synthetic T^2 charts' performances only depend on the Mahalanobis distance, d. The comparative study on the ARL performances of the control charts will identify the better chart in monitoring the process mean vector. The performances of the charts for different sample sizes and correlation coefficients are presented.

The ARL Profiles of the MEWMA Control Chart

Table 1 shows the combinations of optimal parameters of the MEWMA chart derived using the four-step method by Lee and Khoo (2006). This four-step method is as follows:

Step 1 : Specify the smallest acceptable in-control ARL.

Step 2 : Decide on the smallest shift, *d,* that must be detected quickly. For this size of shift, *d*, determine the smoothing constant, *r,* that gives the in-control ARL in Step 1.

Step 3 : Based on the optimal *r* obtained in Step 2, determine the control limit, *H*.

Step 4 : Perform a sensitivity analysis by comparing the out-of-control ARL of the optimal pair (*r*, *H*) to other choices of *r* and *H* that produce the same in-control ARL. Then, select the pair (*r*, *H*) with the most desirable performance, in terms of the out-of-control ARL.

The smallest acceptable in-control ARL of 370, for the design shifts, $d^* \in \{0.5,1.0\}$, is used to derive the smoothing constant, *r*, which is then used to derive the control limit *H*.

TABLE 1
The combination of d^*, *r*, and *H* for $p = 2$ with in-control ARL of 370

Design shifts d^*	Smoothing constant r	Control limit H
0.5	0.045	8.7
1.0	0.13	10.45

The values from Table 1 are then used to compute the ARL values. The ARL profiles for the optimal MEWMA chart when $d^* \in \{0.5,1.0\}$, $n \in \{5,10\}$ and for the correlation coefficients, $\rho \in \{0.2,0.5,0.8\}$ are shown in Tables 3 – 6.

The ARL Profiles of the Multivariate Synthetic Control Chart

Table 2 shows the combinations of the optimal parameters of the synthetic T^2 control chart using the six steps procedure proposed by Ghute and Shirke (2008). The set of (L,UCL) which generated the smallest $ARL_s(d^*)$ is used as the optimal design parameters for the synthetic T^2 control chart. The optimal values for L and UCL are shown for the sample size, $n \in \{5,10\}$ and the design shifts, $d^* \in \{0.5,1.0\}$. These optimal values of (L,UCL) are then used to compute the ARL values for the correlation coefficients, $\rho \in \{0.2,0.5,0.8\}$. The ARL profiles for the optimal synthetic T^2 control chart when $d^* \in \{0.5,1.0\}$ are provided in Tables 3 – 6.

TABLE 2
The optimal design parameters of the synthetic T^2 control chart

Sample size, *n*	Design shifts, d^*	Lower control limit of CRL sub-chart, *L*	Upper control limit of T^2 sub-chart, *UCL*
5	0.5	15	8.52408
	1.0	5	7.47532
10	0.5	8	7.92678
	1.0	2	6.58792

A Comparison of the MEWMA and Multivariate Synthetic Control Charts

A control chart is optimal in detecting a shift if the chart has the smallest out-of-control ARL among all the charts of its kind having the same in-control ARL for a particular shift of interest. From Tables 3– 6, it can be seen that the optimal MEWMA control chart performs better than the optimal synthetic T^2 control chart when the shift, d, is small. For $d < 0.6$, the MEWMA control chart has a better out-of-control ARL performance in detecting the process shift. For moderate shifts, $0.8 \leq d \leq 3.0$, the synthetic T^2 control chart detects an out-of-control signal quicker than the MEWMA control chart. For larger shifts, $d > 3.0$, both the control charts have the same performances in detecting the out-of-control signals.

In the case of the performances of the MEWMA and the synthetic T^2 control charts for different sample sizes, both charts display similar characteristics. Both the control charts perform better in detecting out-of-control signals for sample size $n = 10$ compared to $n = 5$. This is true for both moderate and small shifts, $d < 2.0$. For bigger shifts ($d > 2.0$), the performances of both the charts are independent of the sample size.

There are some similarities and variations in the performances of the charts for different correlation coefficients. Generally, it can be seen from Tables 3 – 6 that as ρ increases, the out-of-control ARL values decrease for both charts. This means that the charts perform better in detecting the out-of-control signals when ρ increases. This is evident especially in the case when $d < 2.0$ for both charts. Similarly, this is also true for $2.0 \leq d \leq 3.0$ for the MEWMA control chart. As for the synthetic T^2 control chart, its performance is independent of the correlation coefficient for moderate shifts, whereas for bigger shifts, the performances of both the charts are independent of the correlation coefficients.

TABLE 3
The ARL profiles for optimal MEWMA and optimal synthetic T^2 control charts when $d^* = 0.5$ and $n = 5$

D	OPTIMAL MEWMA			OPTIMAL SYNTHETIC T^2 CHART		
	$\rho = 0.2$	$\rho = 0.5$	$\rho = 0.8$	$\rho = 0.2$	$\rho = 0.5$	$\rho = 0.8$
0	371.6	371.6	371.6	370.5	370.5	370.5
0.2	36.8	30.7	18.7	177.9	149.5	72.8
0.4	14.6	12.6	8.2	44.6	30.9	10.1
0.6	9.1	7.9	5.4	13.0	9.0	3.3
0.8	6.6	5.8	4.1	5.5	4.0	1.7
1.0	5.3	4.7	3.3	3.1	2.3	1.2
1.5	3.6	3.2	2.2	1.3	1.2	1.0
2.0	2.8	2.4	2.0	1.0	1.0	1.0
2.5	2.2	2.0	1.7	1.0	1.0	1.0
3.0	2.0	2.0	1.1	1.0	1.0	1.0
3.5	2.0	1.8	1.0	1.0	1.0	1.0
4.0	1.8	1.3	1.0	1.0	1.0	1.0
4.5	1.4	1.0	1.0	1.0	1.0	1.0
5.0	1.1	1.0	1.0	1.0	1.0	1.0
5.5	1.0	1.0	1.0	1.0	1.0	1.0
6.0	1.0	1.0	1.0	1.0	1.0	1.0

TABLE 4
The ARL profiles for optimal MEWMA and optimal synthetic T^2 control charts when $d^* = 1.0$ and $n = 5$

D	OPTIMAL MEWMA			OPTIMAL SYNTHETIC T^2 CHART		
	$\rho = 0.2$	$\rho = 0.5$	$\rho = 0.8$	$\rho = 0.2$	$\rho = 0.5$	$\rho = 0.8$
0	368.8	368.8	368.8	368.5	368.5	368.5
0.2	45.6	36.2	18.6	187.2	160.8	81.7
0.4	13.2	10.9	6.5	49.4	35.0	10.6
0.6	7.3	6.2	4.0	14.2	9.2	2.9
0.8	5.0	4.4	2.9	5.3	3.6	1.5
1.0	3.9	3.4	2.4	2.7	2.0	1.1
1.5	2.6	2.3	1.8	1.3	1.1	1.0
2.0	2.0	1.9	1.2	1.0	1.0	1.0
2.5	1.8	1.5	1.0	1.0	1.0	1.0
3.0	1.4	1.1	1.0	1.0	1.0	1.0
3.5	1.1	1.0	1.0	1.0	1.0	1.0
4.0	1.0	1.0	1.0	1.0	1.0	1.0
4.5	1.0	1.0	1.0	1.0	1.0	1.0
5.0	1.0	1.0	1.0	1.0	1.0	1.0
5.5	1.0	1.0	1.0	1.0	1.0	1.0
6.0	1.0	1.0	1.0	1.0	1.0	1.0

TABLE 5
The ARL profiles for optimal MEWMA and optimal synthetic control charts T^2 when $d^* = 0.5$ and $n = 10$

d	OPTIMAL MEWMA			OPTIMAL SYNTHETIC T^2 CHART		
	$\rho = 0.2$	$\rho = 0.5$	$\rho = 0.8$	$\rho = 0.2$	$\rho = 0.5$	$\rho = 0.8$
0	372.3	372.3	372.3	370.6	370.6	370.6
0.2	22.6	19.3	12.1	107.4	82.5	31.4
0.4	9.7	8.4	5.7	16.1	10.9	3.4
0.6	6.2	5.5	3.8	4.4	3.1	1.5
0.8	4.7	4.1	3.0	2.1	1.7	1.1
1.0	3.8	3.4	2.4	1.4	1.2	1.0
1.5	2.6	2.3	2.0	1.0	1.0	1.0
2.0	2.0	2.0	1.3	1.0	1.0	1.0
2.5	2.0	1.8	1.0	1.0	1.0	1.0
3.0	1.6	1.1	1.0	1.0	1.0	1.0
3.5	1.1	1.0	1.0	1.0	1.0	1.0
4.0	1.0	1.0	1.0	1.0	1.0	1.0
4.5	1.0	1.0	1.0	1.0	1.0	1.0
5.0	1.0	1.0	1.0	1.0	1.0	1.0
5.5	1.0	1.0	1.0	1.0	1.0	1.0
6.0	1.0	1.0	1.0	1.0	1.0	1.0

TABLE 6
The ARL profiles for optimal MEWMA and optimal synthetic T^2 control charts when $d^* = 1.0$ and $n = 10$

d	OPTIMAL MEWMA			OPTIMAL SYNTHETIC T^2 CHART		
	$\rho = 0.2$	$\rho = 0.5$	$\rho = 0.8$	$\rho = 0.2$	$\rho = 0.5$	$\rho = 0.8$
0	363.8	363.8	363.8	373.2	373.2	373.2
0.2	23.9	19.2	10.5	123.0	97.9	38.7
0.4	7.8	6.7	1.2	20.7	13.6	3.9
0.6	4.7	4.1	2.8	5.1	3.4	1.4
0.8	3.4	3.0	2.1	2.1	1.6	1.0
1.0	2.7	2.4	1.9	1.3	1.1	1.0
1.5	2.0	1.8	1.1	1.0	1.0	1.0
2.0	1.5	1.2	1.0	1.0	1.0	1.0
2.5	1.1	1.0	1.0	1.0	1.0	1.0
3.0	1.0	1.0	1.0	1.0	1.0	1.0
3.5	1.0	1.0	1.0	1.0	1.0	1.0
4.0	1.0	1.0	1.0	1.0	1.0	1.0
4.5	1.0	1.0	1.0	1.0	1.0	1.0
5.0	1.0	1.0	1.0	1.0	1.0	1.0
5.5	1.0	1.0	1.0	1.0	1.0	1.0
6.0	1.0	1.0	1.0	1.0	1.0	1.0

CONCLUSION

The comparisons made show that the MEWMA control chart performs better than the synthetic T^2 control chart when the shifts are small. However, the synthetic T^2 control chart performs better for moderate shifts. Nonetheless, both the charts perform equally well for larger shifts. The sample sizes and the correlation coefficients have been found to influence the detection of the out-of-control signals. Both charts perform better for larger sample sizes for both small and moderate shifts. However, the performances of both charts are not dependent on the sample sizes for larger shifts. When the correlation coefficient increases, the performances of both charts also improve, especially for small shifts. The performances of both the charts are independent of the correlation coefficient for larger shifts.

There are many areas on the multivariate control charts that are worthy of further research. The construction of a multivariate control chart is usually based on the multivariate normality assumption. Hence, the designs of the multivariate charts for skewed or heavy tailed populations, i.e. when the normality assumption is not fulfilled, are potential topics for further research. In addition, future works can also be conducted to compare the MRL performances of the MEWMA and synthetic T^2 charts to give more meaningful interpretations of the in-control and out-of-control performances of the charts.

REFERENCES

Alkahtani, S., & Schaffer, J. (2012). A double multivariate exponentially weighted moving average (dMEWMA) control chart for a process location monitoring. *Communications in Statistics-Simulation and Computation*, *41*, 238-252.

Aparisi, F., & de Luna, M. A. (2009). The design and performance of the multivariate synthetic T^2 control chart. *Communications in Statistics-Theory and Methods*, 38, 173-192.

Bersimis, S., Psarakis, S., & Panaretos, J. (2007). Multivariate statistical process control charts: an overview. *Quality and Reliability Engineering International*, *23*, 517-543.

Bourke, P. D. (1991). Detecting a shift in fraction nonconforming run-length control charts with 100% inspection. *Journal of Quality Technology*, *23*, 225-238.

Ghute, V. B., & Shirke, D. T. (2008). A multivariate synthetic control chart for monitoring process mean vector. *Communications in Statistics-Theory and Methods*, *37*, 2136-2148.

Lee, M. H., & Khoo, M. B. C. (2006). Optimal statistical design of a multivariate EWMA chart based on ARL and MRL. *Communications in Statistics-Simulation and Computation*, *35*, 831-847.

Lowry, C. A., Woodall, W. H., Champ, C. W., & Rigdon, S. E. (1992). A multivariate exponentially weighted moving average control chart. *Technometrics*, *34*, 46-53.

Khoo, M. B. C., Abdu A. M. A., & Wu, Z. (2009). A multivariate synthetic control chart for monitoring the process mean vector of skewed populations using weighted standard deviations. *Communications in Statistics-Simulation and Computation*, *38*, 1493-1518.

Khoo, M. B. C., & Quah, S. H. (2004). Pengiraan persentil taburan panjang larian bagi carta kawalan purata bergerak berpemberat eksponen multivariat. *Pertanika Journal of Science & Technology*, *12*, 33-43.

Khoo, M. B. C., Wong, V. H., Wu, Z., & Castagliola, P. (2011). Optimal designs of the multivariate synthetic chart for monitoring the process mean vector based on median run length. *Quality and Reliability Engineering International, 27,* 981-997.

Montgomery, D. C. (2009). *Statistical quality control: a modern introduction* (6th Edition). Asia: John Wiley & Sons.

Niaki, S. T. A., & Ershadi, M. J. (2012). A hybrid ant colony, Markov chain, and experimental design approach for statistically constrained economic design of MEWMA control charts. *Expert Systems with Applications*, *39*, 3265-3275.

Prabhu, S. S., & Runger, G. C. (1997). Designing a multivariate EWMA control chart. *Journal of Quality Technology*, *29*, 8-15.

Shewhart, W. A. (1986). *Statistical method from the viewpoint of quality control*, Mineola, NY: Dover publications.

Wu, Z., & Spedding, T. A. (2000). A synthetic control chart for detecting small shifts in the process mean. *Journal of Quality Technology*, *32,* 32-38.

Xie, Y., Xie, M., & Goh, T. N. (2011). Two MEWMA charts for Gumbel's bivariate exponential distribution. *Journal of Quality Technology*, *43*, 50-65.